

The IFF Glossary for the Lattice of Theories

Figure 1 graphically illustrates most of the core classes/sets, relations and functions needed to specify the truth concept lattice and the lattice of theories for any fixed first order logic (FOL) type language L . Those in **red** are only in the truth concept lattice namespace; those in **blue** are only in the lattice of theories namespace; and those in **pink** are in both namespaces. Of the five collections in the picture, the class of models and the set of expressions are basic, the set of theories is the power set of expressions, and the set of closed theories is derived from the set of theories and the closure operator. In addition, there are sixteen functions and ten binary relations. Hence, around thirty directly usable terms are used to specify the lattice of theories and truth concept lattice. The right side of the diagram is the formal aspect. Missing from Figure 1 are the meet and join function and the top and bottom elements. Also missing are the tests for inconsistency and mutual consistency. Some helper terms not included are also used in the axiomatization. A final caveat is that the lattice of theories axiomatization is situated within the richer and more extensive category of theories axiomatization.

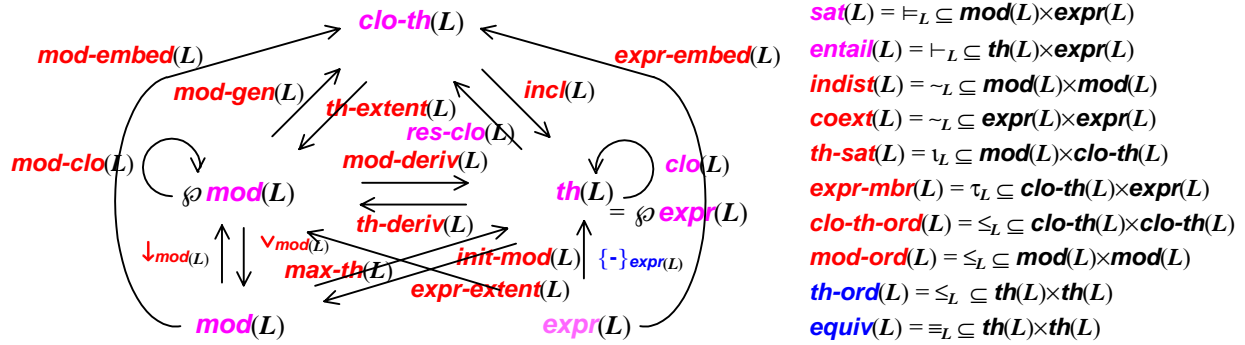


Figure 1: Sets, Functions and Relations in the Interpretative/Formal Aspect of the Lattice of Theories

Primitives	Comments
$mod-deriv(L) : \wp \text{mod}(L) \rightarrow \text{th}(L) = \wp \text{expr}(L)$ $th-deriv(L) : \wp \text{expr}(L) = \text{th}(L) \rightarrow \wp \text{mod}(L)$	These are the derivation functions defined on the truth classification. They are the principle means for defining the truth concept lattice.
$incl(L) : \text{clo-th}(L) \rightarrow \text{th}(L)$	This inclusion function is not truly primitive, since it is dependent upon the definition of closed theories.
$\{-\}_{\text{expr}(L)} : \text{expr}(L) \rightarrow \wp \text{expr}(L) = \text{th}(L)$ $\downarrow_{\text{mod}(L)} : \text{mod}(L) \rightarrow \wp \text{mod}(L)$	These are the standard pointwise embeddings for sets and orders, respectively. The subclass of models is regarded as the collection of down-closed subsets.
$init-mod(L) : \text{th}(L) \rightarrow \text{mod}(L)$	The initial model of a theory is the free model restricted to those tuples that satisfy the theory.

Function Identities	
$mod-clo(L) = mod-deriv(L) \cdot th-deriv(L)$ $clo(L) = th-deriv(L) \cdot mod-deriv(L)$	Closure Operators: These are induced by the derivation adjunction between subclasses of models and theories.
$th-deriv(L) = init-mod(L) \cdot \downarrow_{mod(L)}$ $init-mod(L) = th-deriv(L) \cdot \vee_{mod(L)}$ $max-th(L) = \downarrow_{mod(L)} \cdot mod-deriv(L)$ $mod-deriv(L) = \vee_{mod(L)} \cdot max-th(L)$	
$res-clo(L) \cdot incl(L) = clo(L)$ $expr-extent(L) = \{-\}_{expr(L)} \cdot res-clo(L)$ $mod-gen(L) \cdot incl(L) = mod-deriv(L)$ $th-extent(L) = incl(L) \cdot th-deriv(L)$ $mod-embed(L) = \downarrow_{mod(L)} \cdot mod-gen(L)$ $expr-embed(L) = \{-\}_{expr(L)} \cdot res-clo(L)$ $\vee_{mod(L)} = mod-deriv(L) \cdot init-mod(L)$	
$expr-extent(L) \cdot mod-gen(L) = expr-extent(L)$	

Adjointness Constraints	
$mod-gen(L) \cdot th-extent(L) \supseteq id_{\wp mod(L)}$ $th-extent(L) \cdot mod-gen(L) = id_{clo-th(L)}$	Generation-Extent Adjunction
$incl(L) \cdot res-clo(L) = id_{clo-th(L)}$ $res-clo(L) \cdot incl(L) \supseteq (\equiv) id_{th(L)}$	Theory Equivalence: The lattice of theories is order-theoretically equivalent to the truth concept lattice.
$mod-deriv(L) \cdot th-deriv(L) \supseteq id_{\wp mod(L)}$ $th-deriv(L) \cdot mod-deriv(L) = id_{th(L)} = id_{\wp expr(L)}$	Derivation Adjunction: The derivation adjointness is basic in FCA. It is the composition of the generation-extent adjunction and the theory equivalence.
$max-th(L) \cdot init-mod(L) \equiv id_{mod(L)}$ $init-mod(L) \cdot max-th(L) \supseteq (\equiv) id_{th(L)} = id_{\wp expr(L)}$	Theory-Model Equivalence: It is the composition of the join-principal-ideal adjunction and the derivation adjunction. The initial model is the free model restricted by satisfaction.
$\vee_{mod(L)} \cdot \downarrow_{mod(L)} \supseteq id_{\wp mod(L)}$ $\downarrow_{mod(L)} \cdot \vee_{mod(L)} = id_{mod(L)}$	Join-Principal-Ideal Adjunction: The join operator is definable as the composition of model derivation and initial model.

Equivalence Relations	
$indist(L)$, induced by $max-th(L)$	Two models are indistinguishable when they have the same intent in the truth classification; that is, they generate the same theory.
$coext(L)$, induced by $expr-extent(L)$	Two expressions are coextensive when they have the same extent in the truth classification; that is, they are satisfied by the same models.

Orders	
$clo-th-ord(L)$	Reverse subset inclusion on closed theories.
$th-ord(L)$, induced by $res-clo(L)$	Entailment order, equivalent to the closed theory order.
$mod-ord(L)$, induced by $mod-embed(L)$	Specialization-generalization order on models.

Relational Identities	
$th-sat(L) \circ expr-mbr(L) = sat(L)$	This is a part of the basic theorem of FCA applied to the truth concept lattice.

Terminology

- L

IFF-KIF Code:

language

Meaning:

The symbol L denotes a fixed FOL language.

Classes and Sets

- $mod(L)$

IFF-KIF Code:

(where ‘?L’ denotes the fixed language being used.

(model ?L)

Meaning:

The symbol $mod(L)$ denotes the class of models for the language L . L -models in the IFF provide an interpretive semantics for object-level ontologies.

- $expr(L)$

IFF-KIF Code:

(expression ?L)

Meaning:

The symbol $expr(L)$ denotes the set of expressions for the language L . The set of L -expressions is built up recursively in the IFF.

- $th(L)$

IFF-KIF Code:

(theory ?L)

Meaning:

The symbol $th(L)$ denotes the set of theories for the language L . For a fixed language L , in the IFF the set of theories is identical (not just isomorphic) to the set of all subsets of expressions of L : $th(L) = \wp expr(L)$. Any expression entailed by a theory T is called a theorem of T .

- $clo-th(L)$

IFF-KIF Code:

(closed-theory ?L)

Meaning:

The symbol $clo\text{-}th(L)$ denotes the set of closed theories for the language L . A theory is closed when it equals its closure: $T = clo(L)(T)$. This means that the axiom set of T equals the theorem set of T : $axm(T) = thm(T)$. The set of closed theories represent precisely the set of formal concepts of the truth concept lattice.

Binary Relations

- $sat(L) = \models_L$

IFF-KIF Code:

(satisfaction ?L)

Meaning:

The symbol $sat(L) = \models_L$ denotes the binary satisfaction relation between models and expressions. It is also known as the truth classification relation. An L -model M satisfies (or is a model of) an L -expression e , symbolized by

$$M \models_L e$$

when all tuples of M satisfy e . Satisfaction is the most basic relation of semantics.

- $indist(L) = \sim_L$

IFF-KIF Code:

(indistinguishable ?L)

Meaning:

The symbol $indist(L) = \sim_L$ denotes the binary indistinguishable relation between two models. For any type language L , two L -models M_1, M_2 are indistinguishable

$$M_1 \sim_L M_2$$

when they have the same formal intent (maximal theory): $max\text{-}th(M_1) = max\text{-}th(M_2)$.

- $coext(L) = \sim_L$

IFF-KIF Code:

(coextensive ?L)

Meaning:

The symbol $coext(L) = \sim_L$ denotes the binary coextensive relation between two expressions. For any type language L , two L -expressions e_1, e_2 are coextensive

$$e_1 \sim_L e_2$$

when they have the same interpretive extent (expression intent): $expr\text{-}intent(e_1) = expr\text{-}intent(e_2)$.

- $entail(L) = \vdash_L$

IFF-KIF Code:

(entailment ?L)

Meaning:

The symbol $entail(L) = \vdash_L$ denotes the binary entailment relation between theories and expressions. The principal semantic (and also proof-theoretic) relation on theories is the binary entailment relation between a theory and an expression. A theory T entails an expression e , symbolized

$$T \vdash_L e$$

when any model for (the axioms of) T is also a model for e .

- $clo\text{-}th\text{-}ord(L) = \leq_L$

IFF-KIF Code:

(closed-theory-order ?L)

Meaning:

The symbol $clo\text{-}th\text{-}ord(L) = \leq_L$ denotes the order between two closed theories. For any type language L , two closed L -theories T_1, T_2 are ordered

$$T_1 \leq_L T_2$$

when any T_2 -theorem is a T_1 -theorem; that is, when the second theory is a subset of the first theory: $T_2 \subseteq_L T_1$. Then T_1 is said to be more specialized than T_2 , and T_2 is said to be more generalized than T_1 . This is a true generalization-specialization hierarchy.

- $mod\text{-}ord(L) = \leq_L$

IFF-KIF Code:

(model-order ?L)

Meaning:

The symbol $mod\text{-}ord(L) = \leq_L$ denotes the order between two models. For any type language L , two L -models M_1, M_2 are ordered

$$M_1 \leq_L M_2$$

when the intents (maximal theories) of the two models are covariantly ordered: $max\text{-}th(M_1) \leq_L max\text{-}th(M_2)$.

- $th\text{-}ord(L) = \leq_L$

IFF-KIF Code:

(theory-order ?L)

Meaning:

The symbol $th\text{-}ord(L) = \leq_L$ denotes the order between two theories. For any type language L , two L -theories T_1, T_2 are ordered

$$T_1 \leq_L T_2$$

when any T_2 -axiom is a T_1 -theorem; that is, when the first theory entails every axiom of the second theory: $T_1 \vdash_L e_2$ for every $e_2 \in T_2$; that is, when the closures of the two theories are covariantly ordered: $clo(T_1) \leq_L clo(T_2)$. Then T_1 is said to be more specialized than T_2 , and T_2 is said to be more generalized than T_1 . This is a true generalization-specialization hierarchy.

- $th\text{-}sat(L) = \vDash_L = \models_L$

IFF-KIF Code:

(theory-satisfaction ?L)

Meaning:

The symbol $th\text{-}sat(L) = \vDash_L = \models_L$ denotes the binary satisfaction relation between models and closed theories. An L -model M satisfies (or is a model of) a closed L -theory T , symbolized by

$$M \vDash_L T$$

when it satisfies every theorem of T . This means that M is in the truth extent of T ; that is, $M \in extent_L(T)$. This relation, which is also called *iota*, is induced by the model embedding function and the truth concept lattice order. Theory satisfaction is closed on the right with respect to the truth concept lattice order (reverse inclusion between closed theories).

- $expr\text{-}mbr(L) = \tau_L$

IFF-KIF Code:

(expression-membership ?L)

Meaning:

The symbol $\text{expr-mbr}(L) = \tau_L$ denotes the (opposite) binary membership relation between closed theories and expressions. This relation, which is also called tau, is induced by the expression embedding function and the truth concept lattice order. Theory satisfaction is closed on the right with respect to the truth concept lattice order (reverse inclusion between closed theories).

Functions

- $\text{clo}(L) : \text{th}(L) \rightarrow \text{th}(L)$

IFF-KIF Code:

(closure ?L)

Meaning:

The symbol $\text{clo}(L)$ denotes the closure function on L -theories. The closure of an L -theory T is the set-theoretically larger theory consisting of all theorems of T .

- $\text{max-th}(L)$

IFF-KIF Code:

(maximal-theory ?L)

Meaning:

The symbol $\text{max-th}(L)$ denotes the maximal theory function from L -models to L -theories. The maximal theory of a model is the set of all expressions satisfied by the model: this is the 12-fiber function of the satisfaction relation. The maximal theory is closed. It is the largest theory that the model satisfies. In the lattice of theories, it is the meet of all these theories.

- $\text{init-mod}(L)$

IFF-KIF Code:

(initial-model ?L)

Meaning:

The symbol $\text{init-mod}(L)$ denotes the initial model function from L -theories to L -models. This is left adjoint to the maximal theory function. The initial model of a theory is the restriction of the free model over underlying language to the tuples that satisfy the theory. Since it is the initial model in $\text{th-deriv}(L)$, the subcategory of all models that satisfy the theory, it is the join of all models that satisfy that theory.

- $\text{expr-intent}(L)$

IFF-KIF Code:

(expression-intent ?L)

Meaning:

The symbol $\text{expr-intent}(L)$ denotes the expression intent function from L -expressions to subclasses of L -models. The intent of an expression is the class of all models that satisfy the expression. It is the composition of the expression singleton function and the theory derivation function.

- $\text{mod-deriv}(L)$

IFF-KIF Code:

(model-derivation ?L)

Meaning:

The symbol $\text{mod-deriv}(L)$ denotes the model derivation function from subclasses of L -models to theories (subsets of L -expressions). The derivation of a subclass of models is the set-theoretically largest theory (set of all expressions) that they all satisfy.

- $\text{th-deriv}(L)$

IFF-KIF Code:

(theory-derivation ?L)

Meaning:

The symbol $th\text{-deriv}(L)$ denotes the theory derivation function from theories (subsets of L -expressions) to subclasses of L -models. The derivation of a theory is the subclass of models that satisfy the theory.

- $mod\text{-clo}(L)$

IFF-KIF Code:

(model-closure ?L)

Meaning:

The symbol $mod\text{-clo}(L)$, which denotes the model closure function on subclasses of L -models, is the composition of the model derivation function followed by the theory derivation function. A model M is in the closure of a subclass of models M when M satisfies every expression that all the models in M satisfy.

- $mod\text{-gen}(L)$

IFF-KIF Code:

(model-generation ?L)

Meaning:

The symbol $mod\text{-gen}(L)$ denotes the model generation function from subclasses of L -models to closed theories (closed subsets of L -expressions). The (formal truth concept) generation of a subclass of models is the derivation of that subclass. The model derivation function returns closed theories, and hence factors through the set of closed theories – it is the composition of model generation followed by the inclusion of the set of closed theories into the set of theories

$$mod\text{-deriv}(L) = mod\text{-gen}(L) \cdot incl(L).$$